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Vibration control,
Controllable dampers,
MRF and ERF fluids

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APPLICATION OF RHEOLOGICAL FLUID DAMPERS FOR VIBRATION CONTROL OF BUILDINGS

Vibration problem of the building retrofitted with controllable rheological fluid damper was formulated in the paper. Moreover, the problem of the optimal friction force evaluation was also considered. Horizontal displacements of the model under consideration were caused by foundation motion. In case of the system retrofitted with magnetorheological dampers (MR), the signals generated in displacements' sensors are converted into the current in coil exciting magnetic field in slits where the MRF fluid flows. In case of electrorheological dampers (ER) the electric field is formed between electrodes powered by controllable current generator. The electric or magnetic field provokes fluid polarization in damper, what influences on the value of friction force. The intensity of vibrations was evaluated based on a functional depending on accelerations of certain points of the system. Results of computer simulations were also presented in the paper.

WYKORZYSTANIE TŁUMIKÓW Z CIECZAMI REOLOGICZNYMI DO OGRANICZENIA DRGAŃ BUDYNKÓW

W pracy sformułowano matematyczny opis drgań budynku wyposażonego w sterowane tłumiki z cieczami reologicznymi. Przedstawiono również zadanie optymalnego wyboru sił tarcia. Rozpatrzono model, którego poprzeczne drgania są wzbudzane ruchem fundamentu. W układzie z tłumikami magnetoreologicznymi (MR), sygnały z czujników przemieszczeń są przetwarzane na prądy płynące w cewkach wzbudzających pole magnetyczne w szczelinach, przez które przepływa ciecz MRF. W przypadku tłumików elektroreologicznych (ER), pomiędzy elektrodami, zasilanymi sterowanym źródłem napięcia, wytwarza się pole elektryczne. Pod wpływem pola magnetycznego albo elektrycznego ciecz w sterowanym tłumiku ulega polaryzacji, co wpływa na wartość siły tarcia. Do oceny intensywności drgań przyjęto funkcjonał, którego wartości zależą od przyspieszenia wybranych punktów konstrukcji. Rozważania zilustrowano przykładową symulacją drgań analizowanego budynku.

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1. INTRODUCTION

The most common methods of vibrations' reduction are stiffening, damping and isolation [6]. Thanks to the stiffening the resonance frequency of the structure is shifted beyond the frequency band of excitations. Damping reduces the resonance peaks by dissipating the energy. Finally, isolation prevents the propagation of excitations to sensitive parts of the system. The problem of civil structures exposed to extreme vibrations is a crucial one in case of tall buildings, bridges and masts. Such events provoke additional forces acting on structures. Thanks to the vibration dampers these forces can be reduced. Nowadays, more frequently the systems of controllable dampers are used [1, 4, 5]. A fundamental feature of these systems is the possibility of the friction force control using special control signals [3].

The main goal of this paper is to formulate the mathematical description of mechanical systems retrofitted with controllable dampers. We will analyse a simple model of multi-storey building submitted to vertical ground motion caused by earthquake [2]. The evaluation of vibration intensity will be carried out using a functional depending on accelerations of certain points. The values of these accelerations depend on the friction forces. Thus, it is possible to establish optimal forces in order to reduce the vibrations.

2. RHEOLOGICAL FLUID DAMPERS

Figure 1 visualizes the scheme of magnetorheological damper (MR) and electrorheological damper (ER). The MR damper contains magnetorheological fluid (MRF) which is polarizated in magnetic field. The magnetic field in MR damper occurs in the slit between the piston and the casing (Fig. 1a). This field is induced by the coil powered by controllable current generator. In case of electrorheological dampers (ER) the electric field is formed between electrodes powered by controllable current generator (Fig. 1b).

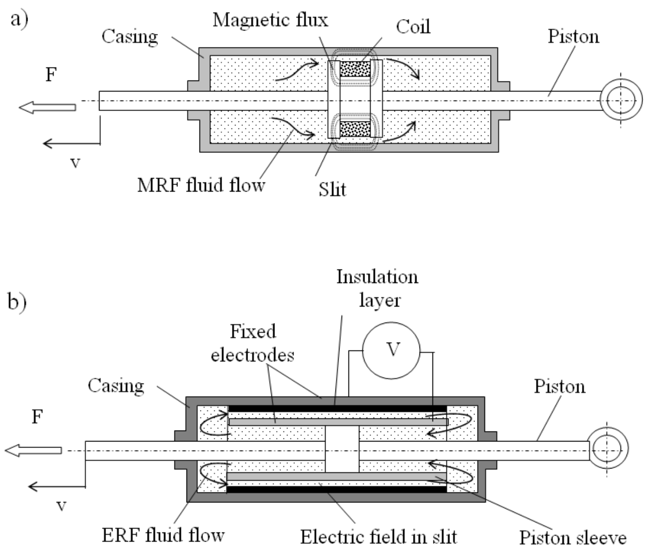

Fig.1. Magnetorheological fluid damper (a) and electrorheological fluid damper (b)

The electric or magnetic field provokes fluid polarization in damper. The polarized MRF or ERF fluid has the property of the viscoplastic material which rheological scheme is shown in Fig. 2, where $\tau_{\lim}$ denotes the limit stress in fluid and μ denotes dynamic viscosity. The polarization process results in increasing of the plastic limit stress along with increases of magnetic of electric field intensity being induced in the slit. Dissipative properties of MRF or ERF fluid can be described as follows

$$\tau = \tau_o(p) + \mu\dot{\gamma}, \quad \tau_o(p) \in \left[-\tau_{\lim}(p), +\tau_{\lim}(p)\right], \tag{1}$$

where

$\dot{\gamma}$ - shear strain rate,

p - the quantity defining the polarization force $p \in [0, 1]$,

$\tau_{\lim}$ - plastic limit in fluid such as $\tau_{\lim}(0) = 0$.

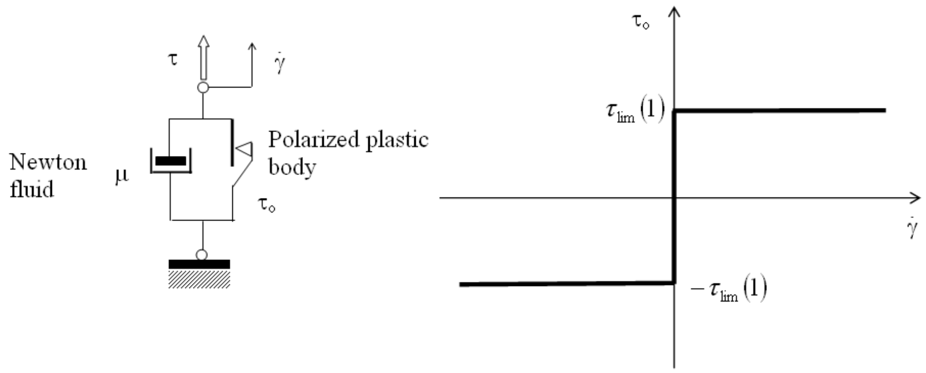

Fig.2. Rheological structure of viscoplastic fluid

Figure 3 contain schematic graph visualizing the above defined characteristics of rheological fluid.

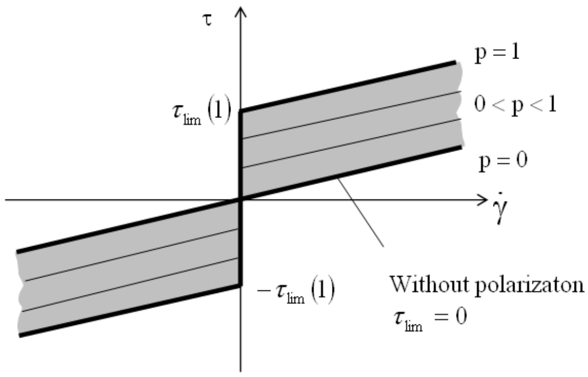

Fig.3. Dissipative characteristics of MRF or ERF fluid

3. DESCRIPTION OF THE MODEL

A mechanical system being a model of a three-storey building is shown in Fig. 4. It is composed of three bodies m_1, m_2, m_3 modelling floors and columns having the stiffness coefficients k_1, k_2, k_3 and three dampers with controllable force. We will assume that the system is retrofitted with MR dampers.

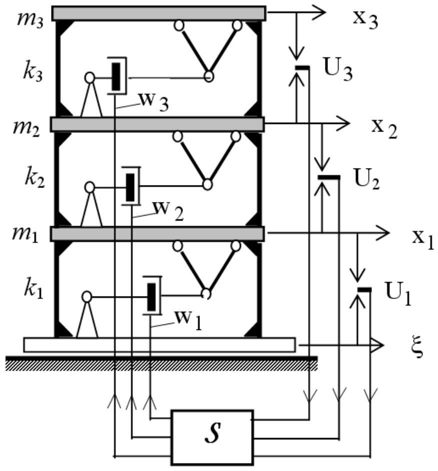

Fig. 4. Model of the building with controllable dampers; U_1, U_2, U_3 – relative displacements' sensors; $\mathcal{S}$ - signal processing system; w_1, w_2, w_3 – dampers' control signals

The vertical motion of the slab floors is described by coordinates x_1, x_2, x_3 defining the displacements with respect to the ground. The function ξ describes the given foundation motion provoking vibrations. There are dampers being installed between the slab floors. The dampers' force depends on relative velocities of floors.

The following definitions hold for the analysed system

$$X := (x_1, x_2, x_3), \quad X_R := (\xi, x_1, x_2, x_3), \quad M := \mathrm{diag}(m_1, m_2, m_3), \tag{2a}$$

where X – vector of coordinates, X_R - extended vector of coordinates , M – mass matrix;

$$U := \begin{bmatrix} x_1 - \xi \\ x_2 - x_1 \\ x_3 - x_2 \end{bmatrix} = H_R^T X_R , \quad H_R^T := \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \; H^T := \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}. \tag{2b}$$

U – vector of relative displacements of floors;
H_R – extended configuration matrix of the system;

$$V := \begin{bmatrix} \dot{x}_1 - \dot{\xi} \\ \dot{x}_2 - \dot{x}_1 \\ \dot{x}_3 - \dot{x}_2 \end{bmatrix}, \quad V = H_R^T \dot{X}_R \,, \quad \dot{X}_R := \left[\dot{\xi}, \dot{x}_1, \dot{x}_2, \dot{x}_3\right]^T , \tag{2c}$$

where V – vector of relative velocities of floors;

$$K := diag\left(k_1, k_2, k_3\right); \qquad S := KU = KH_R^T X_R \qquad T = \left[T_1, T_2, T_3\right]^T , \tag{2d}$$

where K – stiffness matrix; S – vector of elastic forces; T – vector of friction forces;

$$\Omega(V) := \left\{T : T_i \in F(V_i; w), \quad w \in [0,1], \quad i = 1,2,3\right\}, \tag{2e}$$

where $\Omega(V)$ - the set of admissible friction forces, i.e. $T \in \Omega(V)$; the form of this set can be established based on function F ; T_i, V_i - components of the friction force vector and relative velocities.

Using the above introduced definitions we can write the equation of motion of the analysed system

$$M\ddot{X} + HT + HS = 0 \,, \tag{3a}$$

$$T \in \Omega(V) \tag{3b}$$

$$S = KH_R^T X_R = KU \,, \quad V = H_R^T \dot{X}_R \,, \quad X_R := \left[\xi, X^T\right]^T . \tag{3c}$$

The problem being presented above does not possess any solution because the values of friction forces T are defined by multi-valued mapping. In order to find the solution of the problem (3), we need to evaluate an optimal vector T from the set $\Omega(V)$. Thus, we will able to replace the inclusion (4b) by the following problem

$$T = \mathbf{T}_{opt}(S, V) \in \Omega(V) , \tag{4}$$

where $\mathbf{T}_{opt}$ denotes a function of the optimal friction force vector for the system defined by the vectors of relative displacements U and velocities V (see Eqs. (2)). The method of the optimal friction force vector evaluation will be presented below.

Let us assume that the intensity of the system vibration can be evaluated based on the norm of generalized accelerations

$$\ddot{X} = -M^{-1} H(T + S), \quad T \in \Omega(V) . \tag{5}$$

Taking the norm of Eq. (5) leads to the following expressions

$$\frac{1}{2}\left\|\ddot{X}\right\|^2 = \frac{1}{2}(T+S)^T H^T M^{-2} H (T+S), \quad T \in \Omega(V). \tag{6}$$

The right-hand side of Eq. (6) may be treated as a functional of vibration intensity depending on the vector of friction forces T which can be reformulated as follows

$$\text{☞}(T;S) := \frac{1}{2}T^T A T + S^T A T + \frac{1}{2}S^T A S\,, \tag{7}$$

where $A := H^T M^{-2} H$, $S := KU$.

Using the functional $\text{☞}(T;S)$ and the set $\Omega(V)$ we can formulate a problem of the optimal friction force evaluation.

Find the vector $\mathbf{T}_{opt}$ *minimizing the functional* ☞ *in the set* Ω :

$$\mathbf{T}_{opt}(S,V) = \arg\min_{T\in\Omega(V)} \text{☞}(T;S). \tag{8}$$

The solution of the problem (8) enables to find an optimal friction force vector in the system being defined by the vectors of relative displacements and velocities.

4. VIBRATION ANALYSIS OF A BUILDING

The vibrations of the building model retrofitted with controllable dampers can be described by the following system of equations

$$M\ddot{X} + H(T+S) = 0\,, \tag{9a}$$

$$\theta\dot{w} + w = w_{opt}\,, \tag{9b}$$

$$T = F(V,w)\,, \tag{9c}$$

$$w_{opt} = \Phi(T_{opt},V)\,, \tag{9d}$$

$$T_{opt} = \arg\min_{T\in\Omega(V)} \text{☞}(T;U)\,, \tag{9e}$$

where Eq. (9a) describes the motion of the system, Eq. (9b) defines the evolution of the control signal, Eq. (9c) gives the formula for friction force based on the damper's characteristic, Eq. (9d) is the formula for the control signal associated with the optimal friction force, Eq. (9e) describes the problem of the optimal friction force.

Let us analyze a numerical example of a 10-storey building having the following parameters

$$m_i = m = 10^4\ [\text{kg}]\,, \quad k_i = k = 1{,}5\cdot 10^6\ [\text{Nm}^{-1}]\,, \quad i = 1,\ldots,10. \tag{10}$$

Natural vibrations of this system are as follows: {0,3; 0,895; 1,47; 2,01; 2,51; 2,95; 3,26; 3,63; 3,85; 3,98} [Hz]. The foundation motion is defined by the function $\xi(t)=\xi_o \sin(2\pi\omega t)$, having the amplitude $\xi_o = 0{,}01\,[\mathrm{m}]$ and the frequencies shown in Table 1. Time characteristic of the control system (time delay) equals $\theta = 10$ [ms] (see Eq. 9b).

The characteristic of controllable dampers is presented in Fig. 5. Moreover, as a comparison the system with structural damping (without control) was also considered. The property of structural damping is defined by the line OA in Fig. 5. The multi-valued function given in Fig. 5 defines magnetorheological dampers being used for civil structures.

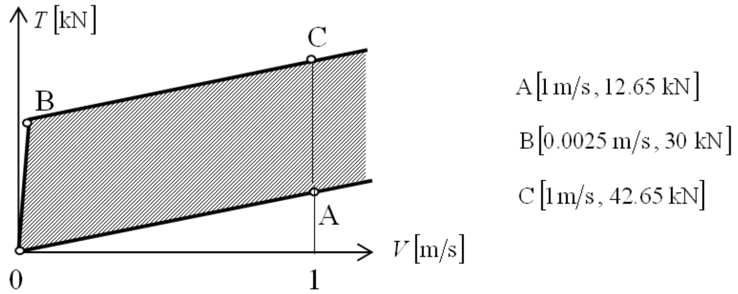

Fig. 5. Diagram of controllable damper used for simulations and coordinates of characteristic points

The following functional was assumed for the vibration intensity evaluation for $[0,t_k)$, $t_k = 6\,[\mathrm{s}]$:

$$\mathbf{a} := \frac{1}{g t_k}\int_0^{t_k} \left\| \ddot{X}(t) \right\|_2 dt \, . \tag{11}$$

Equation (11) defines a medium acceleration norm with respect to the gravitational acceleration *g*. The results of numerical investigations are presented in Fig. 6.

5. CONCLUSIONS

The results shown in Fig. 6 present the dependence of the medium acceleration norm on the frequency of excitation. It is proved analyzing this graphs that the application of controllable dampers may reduce accelerations when the value of excitation frequency is closed to the natural vibration characteristic of the system. Increasing the excitation frequency can make this effect smaller. It should be emphasized that the results being obtained strongly depend on the damper’s characteristic. Our further investigations will deal with the evaluation of the optimal dampers’ properties. Another problem we want to take into consideration is the analysis of various indicators used for vibration intensity evaluation.

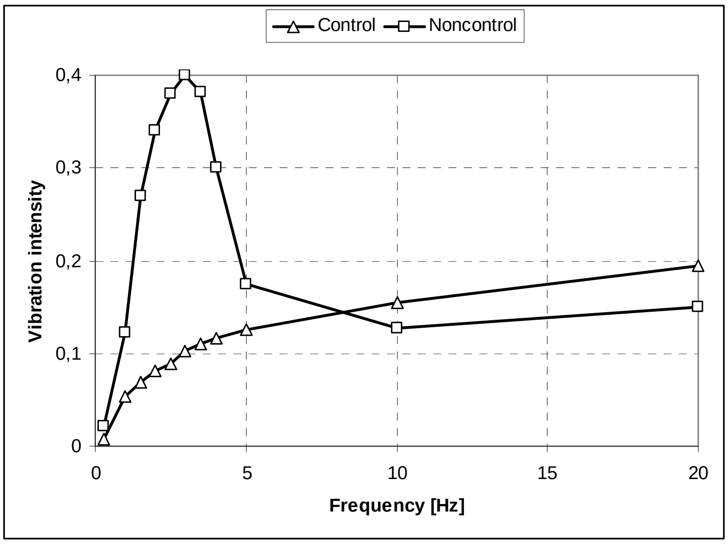

*Fig. 6. Values of the functional **a** based on Eq. 11*

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