Asphalt-aggregate mixtures,
Rheological models,
Fractional derivatives

Artur ZBICIAK[1]
Wiesław GRZESIKIEWICZ[2]

CHARACTERISTICS OF FRACTIONAL RHEOLOGICAL MODELS OF ASPHALT-AGGREGATE MIXTURES

The procedure of the formulation of constitutive equations for asphalt-aggregate mixtures is based very often on linear rheological schemes composed of classical elastic and viscous elements. The parameters of these schemes can be obtained based on laboratory experiments. It was proved in the literature that it is possible to obtain better curve fitting results using non-classical viscoelastic elements described by fractional derivatives. In this paper we will present the characteristics of the fractional Huet-Sayegh model. The problem of estimation of its parameters was analyzed in our previous paper [2]. We will show the results of calculations in the form of creep and relaxation curves as well as hysteretic loops. The characteristics of the fractal model will be compared with the characteristics of the classical viscoelastic Burgers model. The results were obtained using algorithms of numerical calculation of inverse Laplace transforms.

CHARAKTERYSTYKI FRAKTALNYCH MODELI REOLOGICZNYCH MIESZANEK MINERALNO-ASFALTOWYCH

W procedurze formułowania relacji konstytutywnych mieszanek mineralno-asfaltowych (MMA) są wykorzystywane najczęściej liniowe struktury reologiczne zawierające klasyczne elementy sprężyste i lepkie. Wartości parametrów takich struktur wyznacza się na podstawie wyników badań doświadczalnych. W wielu pracach wykazano, iż zastosowanie nieklasycznych elementów lepkosprężystych, opisywanych za pomocą pochodnej ułamkowego rzędu (pochodnej fraktalnej), umożliwia lepsze dopasowanie wyników badań. W niniejszym opracowaniu zajmujemy się charakterystykami fraktalnego modelu Hueta-Sayegha. Zagadnienie estymacji jego parametrów przedstawiliśmy we wcześniejszej pracy [2]. Podamy wyniki obliczeń w postaci krzywych pełzania, relaksacji i pętli histerezy. Charakterystyki modelu fraktalnego zostaną przedstawione na tle charakterystyk klasycznego, lepkosprężystego modelu Burgersa. Rozwiązania uzyskano przy zastosowaniu algorytmów numerycznego wyznaczania odwrotnych transformat Laplace'a.

[1] Warsaw University of Technology, Faculty of Civil Engineering, POLAND; Warsaw 00-637;
16 Armii Ludowej Ave. Phone: +48 22 234-63-49, Fax: +48 22 825-89-46, E-mail: a.zbiciak@il.pw.edu.pl
[2] Warsaw University of Technology, Faculty of Automotive and Construction Machinery Engineering, POLAND; Warsaw 02-524; 84 Narbutta Str., Phone: +48 22 849-05-34, Fax: +48 22 849-03-06,
E-mail: wgr@simr.pw.edu.pl

1. INTRODUCTION

Asphalt-aggregate mixtures constitute basic component for the road pavements construction. The mechanistic design procedures are based on the analysis of stresses and strains in critical points of the structure. Having calculated these values one can evaluate the fatigue resistance based on empirical formulas. The fundamental problem in the procedure of pavement design is to elaborate the appropriate constitutive model suited for the structural behaviour modelling within wide range of mechanical and environmental loadings.

Constitutive equations for asphalt-aggregate mixtures are formulated very often using classical rheological models consisting of springs and dashpots [4]. The simplest models are the Kelvin-Voigt and Maxwell models consisting of spring and dashpot in parallel and in series respectively. A more accurate model of viscoelastic behaviour for asphaltic materials is the Burgers model, which consists of the Maxwell model combined in series with the Kelvin-Voigt model. It is possible to express the constitutive equation of viscoelastic materials in terms of fractional order derivatives of stress and strain. Such formulation leads to so called fractional rheological models. Applying this formulation it is possible to obtain better curve fitting results in the procedure of parameters' evaluation based on experimental data.

In this paper we will present the characteristics of the fractional Huet-Sayegh model. The problem of estimation of its parameters was analyzed in our previous paper [2]. We will show the results of calculations in the form of creep and relaxation curves as well as hysteretic loops. The characteristics of the fractional model will be compared with the characteristics of the classical viscoelastic Burgers model.

2. LINEAR RHEOLOGICAL ELEMENTS

Rheological properties of viscoelastic materials may be modelled applying the stress relaxation function G, which determines the time history of the stress excited by unit step change in strain [1, 5]. Thus, the relaxation function is a step characteristic of rheological scheme. Applying the relaxation function one can define an integral operator assigning the stress function σ to the differentiable strain function ε what leads to the following equation

$$\sigma(t) = G(t)\varepsilon(0) + \int_0^t G(t-\tau)\dot{\varepsilon}(\tau)\,\mathrm{d}\tau \tag{1}$$

The above equation represents the integral form of the constitutive equation for a viscoelastic solid. The equation is also referred to as a hereditary or convolution integral.

Let us assume the following stress relaxation function defining so called fractional rheological element [8]

$$G(t) := \eta \frac{1}{\Gamma(1-\alpha)t^{\alpha}}\ ; \quad \alpha \in (0,1) \tag{2}$$

where η denotes a material parameter and Γ denotes Gamma function

$$\Gamma(1-\alpha) := \int_0^\infty t^{-\alpha} e^{-t} \mathrm{d}t \tag{3}$$

Constitutive relationships of the fractional rheological element may be expressed in differential form

$$\sigma(t) := \eta \mathrm{D}^\alpha \varepsilon(t); \quad \alpha \in (0,1) \tag{4}$$

where $\mathrm{D}^\alpha \equiv \frac{\mathrm{d}^\alpha}{\mathrm{d}t}$ denotes the α -th derivative operator having the following form [3, 6]

$$\mathrm{D}^\alpha \varepsilon(t) := \frac{\varepsilon(0)}{\Gamma(1-\alpha)\, t^\alpha} + \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\dot{\varepsilon}(\tau)}{(t-\tau)^\alpha} \mathrm{d}\tau \tag{5}$$

The creep compliance function of the fractional element is defined as follows

$$J(t) := \frac{t^\alpha}{\eta \Gamma(1+\alpha)}; \quad \alpha \in (0,1) \tag{6}$$

Comparative visualization of elastic, fractional and viscous elements is shown in Fig. 1. Along to the graphical symbols the operator descriptions are given based on Eq. (4).

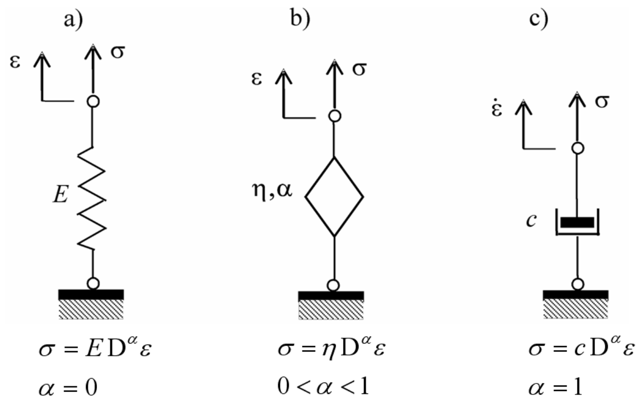

Fig.1. Comparative visualization of elastic (a), fractional (b) and viscous (c) elements.

The constitutive description of the fractional element shows that this element exhibits both elastic and viscous properties depending on the value of $\alpha \in (0,1)$. For $\alpha \to 0$ the elastic properties are dominant while in case of $\alpha \to 1$ the element behaves like a viscous dashpot.

Figure 2 presents two viscoelastic rheological models suited for constitutive modelling of asphalt-aggregate mixtures – classical Burgers model and fractional Huet-Sayegh model. Our previous paper [2] was devoted to the problem of estimation of their parameters based on experiments. In case of the Burgers model the following results were obtained: $E_1 = 12446\,\mathrm{MPa}$, $E_2 = 7195\,\mathrm{MPa}$, $c_1 = 368\,\mathrm{MPa \cdot s}$ and $c_2 = 126\,\mathrm{MPa \cdot s}$. For the Huet-

Sayegh model the procedure of the curve fitting leads to the values as follows: $E_1 = 1003 \cdot 10^5\,\mathrm{MPa}$, $E_2 = 0\,\mathrm{MPa}$, $\alpha_1 = 0.787$, $\alpha_2 = 0.247$, $\eta_1 = 660\,\mathrm{MPa} \cdot \mathrm{s}^{\alpha_1}$, $\eta_2 = 2985\,\mathrm{MPa} \cdot \mathrm{s}^{\alpha_2}$. It was proved that using the fractional model one can better fit the experiments within wide range of the excitation frequencies.

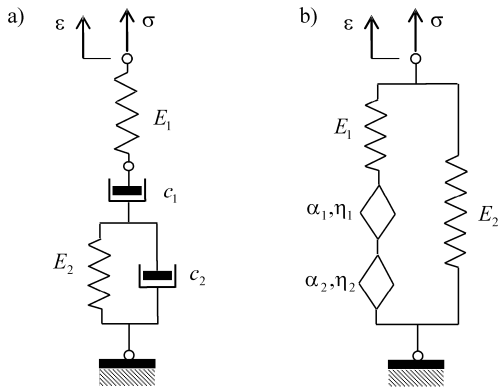

Fig. 2. Classical Burgers model (a) and fractional Huet-Sayegh model (b).

3. NUMERICAL RESULTS

In order to establish the characteristics of rheological models shown in Fig. 2, we will use their transfer functions. In case of Burgers model we have the following equation

$$E_B^*(s) = \frac{1}{\frac{1}{E_1} + \frac{1}{s c_1} + \frac{1}{E_2 + s c_2}} \tag{7}$$

while the Huet-Sayegh transfer function is as follows

$$E_H^*(s) = E_2 + \frac{1}{\frac{1}{E_1} + \frac{1}{\eta_1 s^{\alpha_1}} + \frac{1}{\eta_2 s^{\alpha_2}}} \tag{8}$$

For linear systems analyzed in this paper we can write the following equation relating stresses and strains

$$\sigma^*(s) = E^*(s)\,\varepsilon^*(s) \tag{9}$$

where $\sigma^*(s)$ and $\varepsilon^*(s)$ denote Laplace transforms of the stress $\sigma(t)$ and strain $\varepsilon(t)$ states respectively. Having analytical forms of the Laplace transforms for excitations suited for evaluation of such characteristics as creep/relaxations curves and hysteretic loops one can find the solution using inverse Laplace transform. This operation will be carried out numerically using algorithms described in [7].

The results of numerical calculations are shown in Fig. 3÷5. It shows the differences in characteristics for the classical Burgers model and fractional Huet-Sayegh model.

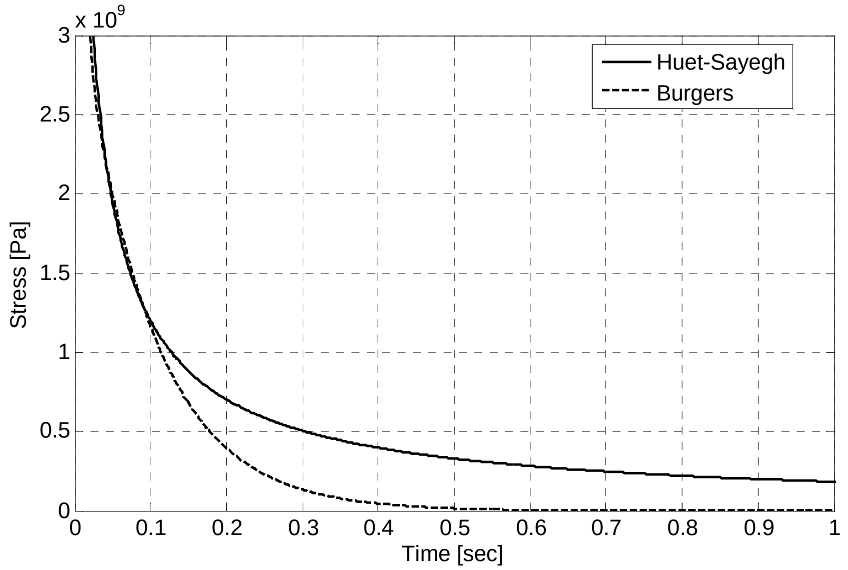

Fig. 3. Stress relaxation curves for Burgers and Huet-Sayegh models.

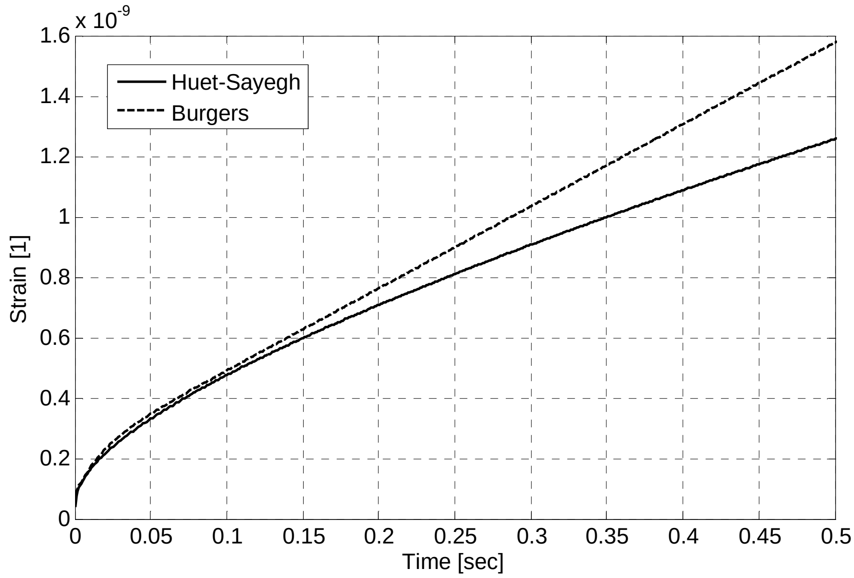

Fig. 4. Creep compliance behaviour of Burgers and Huet-Sayegh models.

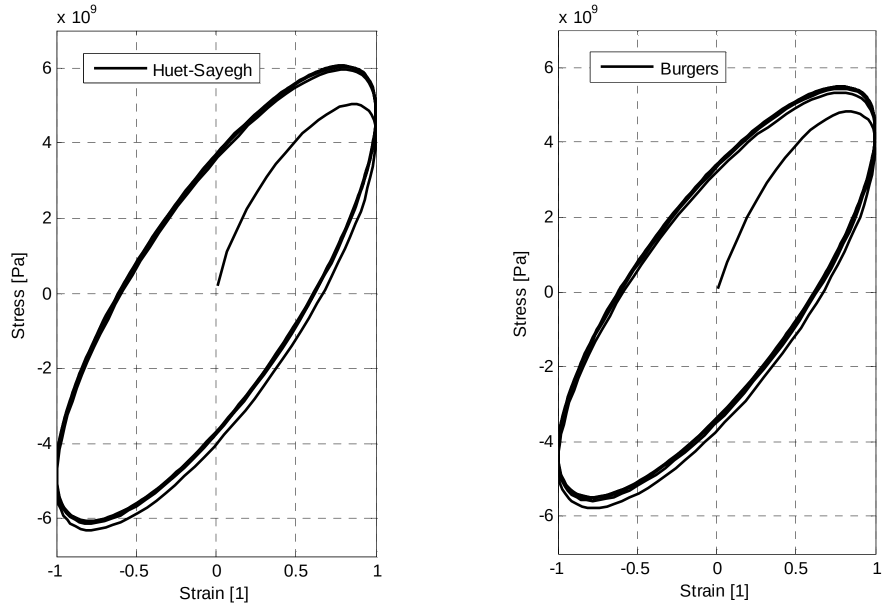

Fig. 5. Hysteretic loops of Huet-Sayegh and Burgers models (strain excitation).

4. FINAL REMARKS

The majority of methods used for numerical calculation of inverse Laplace transforms have serious limitations concerning the class of functions that can be inverted or the achievable accuracy. The procedures applied in this paper can be used for analysis of fractional rheological models. The required accuracy of the results can be enhanced without changing the algorithm, only at the cost of a longer computation time.

5. REFERENCES

[1] Bland D. R.: *The Theory of Linear Viscoelasticity*. Pergamon Press, Oxford 1960.

[2] Grzesikiewicz W., Zbiciak A.: *Zastosowanie pochodnej ułamkowego rzędu do modelowania mieszanek mineralno-asfaltowy*ch. Pomiary Automatyka Kontrola, 9/2011, s. 1048-1051.

[3] Kaczorek T.: *Wybrane zagadnienia teorii układów niecałkowitego rzędu*. Oficyna Wydawnicza Politechniki Białostockiej, Białystok 2009.

[4] Kim Y.R.: *Modeling of Asphalt Concrete*. ASCE Press, McGraw-Hill, New York 2009.

[5] Nowacki W. *Teoria pełzania*. Arkady, Warszawa 1963.

[6] Podlubny I.: *Fractional Differential Equations*. Mathematics in Science and Engineering Vol. 198, Academic Press, 1999.

[7] Valsa J., Brancik L.: *Approximate formulae for numerical inversion of Laplace transforms.* Int. J. Numer. Model. 11, 153–166, 1998.

[8] Zbiciak A.: *Dynamika materiałów i konstrukcji o nieklasycznych charakterystykach sprężysto-dyssypacyjnych*. Prace Naukowe Politechniki Warszawskiej, Budownictwo z. 152, s. 1-156, OWPW, Warszawa 2010.