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BLDC motor,
angular velocity control

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ANGULAR VELOCITY CONTROL OF INVERTER DRIVE SYSTEM WITH THE BRUSHLESS DC MOTOR

The problem of drive system velocity regulation with brushless dc motor was analyzed. Motor is supplied by the voltage inverter. For parametrical identification of the drive system mathematical model and selection of controller parameters, the Box's numerical static optimization method was used. A subject of analysis was the sensitivity of control system structures to variations of motor parameter values. Laboratory tests were carried out in the system with 2 kW motor.

REGULACJA PRĘDKOŚCI KĄTOWEJ NAPĘDU Z SILNIKIEM BEZSZCZOTKOWYM ZE WZBUDZENIEM OD MAGNESÓW TRWAŁYCH

Analizowano problem regulacji prędkości układu napędowego z silnikiem bezszczotkowym prądu stałego z komutatorem elektronicznym. Silnik zasilano z falownika napięcia. Do identyfikacji parametrycznej modelu matematycznego napędu i doboru parametrów regulatora zastosowano numeryczną metodę optymalizacji statycznej Box'a. Analizowano wrażliwość struktur układów sterowania na zmiany wartości parametrów silnika. Badania laboratoryjne przeprowadzono w układzie napędowym z silnikiem o mocy 2 kW.

1. INTRODUCTION

So far, in electric drive systems, mainly two types of motors have been used, that is, the induction or direct-current classic ones. Nowadays, more and more commonly, the new generations of motors are used including motors with permanent magnets or synchronic reluctance motors. Slight power losses, small overall dimensions and lowering prices result in a situation when motors with permanent magnets are now competing with induction motors.

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Motors with permanent magnets, in relation to induction motors, are characterized with higher efficiency, higher power achieved from the mass unit, momentum high torque overload capacity and very good regulative parameters.

There are two main types of motors with permanent magnets [1, 5]:

– motors with trapezoidal field distribution in a gap (*Brushless DC Motor or BLDC)*,
– motors with sinusoidal field distribution in a gap (*Permanent Magnet Synchronous Motor or PMSM*).

BLDC motors are characterized by slightly higher torque pulsation, but less complicated control system.

In further discussion, for the purpose of drive analysis, a brushless dc motor was used (BLDC). A measurement of motor physical quantities and control was carried out with the use of a computer and ADMC 401 microprocessor system using the a/d and d/a converter card.

A subject of this paper is the analysis of the problem of parametrical identification of motor mathematical model, principles of selection of controller parameters in the angular velocity regulation system and analysis of this system applying to variations of drive parameter values.

2. MOTOR MATHEMATICAL MODEL

Principles of operation and basic features are widely described in [1, 2, 5]. Motor electric diagram is presented in fig. 1. Control of transistors is dependent on rotor orientation angle θ. This angle is usually measured with the use of three Hall sensors arranged symmetrically on rotor perimeter.

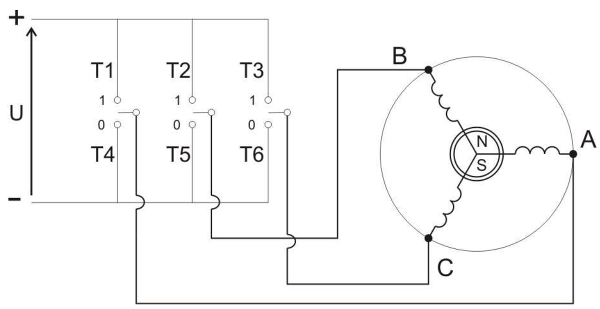

Fig. 1. Electric diagram of three-phase BLDC motor

Most common algorithm for the control of BLDC engines is, so called classic algorithm. It relies on cyclic switching over between inverter transistors, resulting only from a rotor position (fig. 2). In fig. 2, A, B and C mean particular motor winding phases, whereas H are the upper (positive) group transistors, and L - the lower (negative) group transistors.

Electrical circuit equations have the following form [4]

$$L_S \frac{d}{dt} i_i = -R_S\, i_i - E_i + v_i \tag{1}$$

where: R_s – stator phase winding resistance ; L_s – stator phase winding inductance; i_i – i-th phase current; v_i – i-th phase voltage; E_i – EMF induced in the i-th phase.

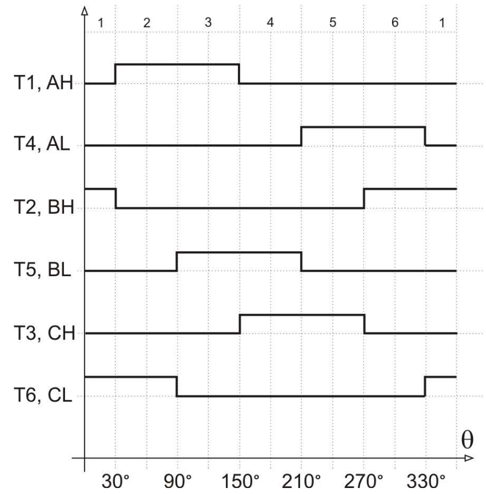

Fig. 2. Signals controlling transistors in function of the angle θ of rotor shaft

Electromotive force can be described in the following form

$$E_i = \omega K_i \tag{2}$$

where K_i – induction coefficient. This coefficient depends on the rotation angle of rotor shaft (fig. 3). For the i–th ideal motor phase it has a trapezoidal shape, and for conductive transistors it is constant.

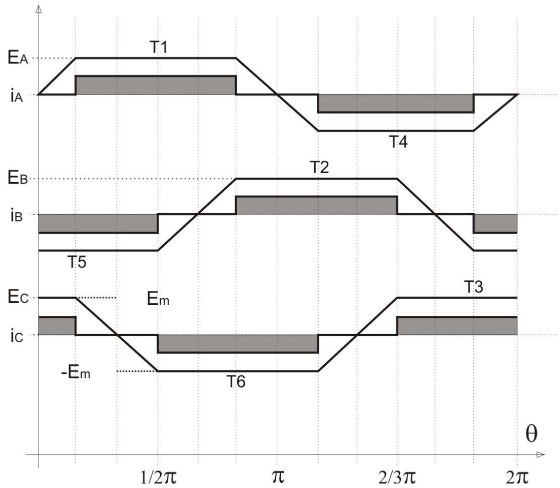

Fig.3. Time courses of phase currents (i_A, i_B, i_C) and EFM (E_A, E_B, E_C)

Induction coefficient K_A, for rotor angle θ variations within the range from 0 to 2π, can be defined with the following equations:

$$\begin{array}{lll} \frac{6K_m}{\pi}\theta & \text{for} & \theta \in \left(0; \frac{\pi}{6}\right) \\ K_m & \text{for} & \theta \in \left(\frac{\pi}{6}; \frac{5\pi}{6}\right) \\ -\frac{6K_m}{\pi}(\theta - \pi) & \text{for} & \theta \in \left(\frac{5\pi}{6}; \frac{7\pi}{6}\right) \\ -K_m & \text{for} & \theta \in \left(\frac{7\pi}{6}; \frac{11\pi}{6}\right) \\ \frac{6\mathrm{K_m}}{\pi}(\theta - 2\pi) & \text{for} & \theta \in \left(\frac{11\pi}{6}; 2\pi\right) \end{array} \tag{3}$$

where K_m is the constant value of induction coefficient (in the time of added transistors). Other coefficients, that is, K_B and K_C, are out of phase respectively by $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. To make use of equations (3) within the $\theta \in (0\,; 2\pi)$ set, to determine the K_B, the current rotor position angle θ, should be replaced with θ_1 angle, where:

$$\begin{array}{lll} \theta_1 = \theta + 2\pi - \frac{2\pi}{3} & \text{for} & \theta < \frac{2\pi}{3} \\ \theta_1 = \theta - \frac{2\pi}{3} & \text{for} & \theta \geq \frac{2\pi}{3} \end{array} \tag{4}$$

and respectively for coefficient K_C:

$$\begin{array}{lll} \theta_2 = \theta + 2\pi - \frac{4\pi}{3} & \text{for} & \theta < \frac{4\pi}{3} \\ \theta_2 = \theta - \frac{4\pi}{3} & \text{for} & \theta \geq \frac{4\pi}{3} \end{array} \tag{5}$$

Mechanical and motor electromagnetic torque equations are defined as with the following:

$$J\frac{d\omega}{dt} = M - M_o\ , \qquad M = \frac{1}{\omega}\left(E_A i_A + E_B i_B + E_C i_C\right) = K_A i_A + K_B i_B + K_C i_C \tag{6}$$

where: J – inertia moment of motor; M_o – load torque.

Input signals in mathematical model (1)–(6) are stator phase voltage, whereas the output signals – angular velocity ω, position θ of rotor shaft and phase currents. Phase voltages can be registered by direct measurement, i.e. with the use of Hall sensors. Usually the inverter's rectified voltage is measured, and voltages are determined basing on signals from rotor position sensors. Whereas, instead of phase currents, for the purpose of identification, a stator current vector amplitude can be used.

Inductance L_s and motor induction coefficient K_m was determined according to minimization of the mean-square error of the stator current I

$$Q = \sum_{i=1}^{N} \left(I(i) - \hat{I}(i) \right)^2 , \qquad I = \sqrt{i_\alpha^2 + i_\beta^2} \tag{7}$$

where: N – number of measurements, i_α, i_β – two-phase components of stator current, and with the symbol "^" the solution of motor mathematical model was defined. In the identification process, inductance L_s, induction coefficient K_m and moment of inertia J were determined. Stator resistance R_s was determined by direct measurement.

Whereas the moment of inertia J is determined from a minimization of the function

$$Q = \frac{1}{N} \sum_{i=1}^{N} \left(\omega(i) - \hat{\omega}(i) \right)^2 \tag{8}$$

As a result of the identification of motor mathematical model on the basis of a transient state, (motor was excited by step change of stator voltage value of 52,3 V) and minimization of function (7) and (8) – values of parameters listed in table 1 were determined. Stator resistance was determined by direct measurement (R_s=1,09 Ω).

Tab. 1. Identification results

Etap	L_s	K_m	J
1	0,0061	0,478	
2			0,0639

In fig. 4, registered time responses are listed stator current amplitudes I and its mathematical model, for values of parameters determined during a process of function minimization (7).

Fig. 5 shows the results of identification as a result of function minimization (8).

3. VELOCITY REGULATION SYSTEM

Input quantity of BLDC motor is the three-phase voltage amplitude (in case of three-phase motor). Variable value of this amplitude is achieved by modulation of rectified three-phase voltage. Time responses of angular velocity and motor torque for excitation voltage amplitude equal 60 V are shown in fig. 6. Characteristic feature of BLDC motors is a torque pulsation. Angular velocity or motor torque control in open system can be carried out in case of systems with non-excessive quality requirements.

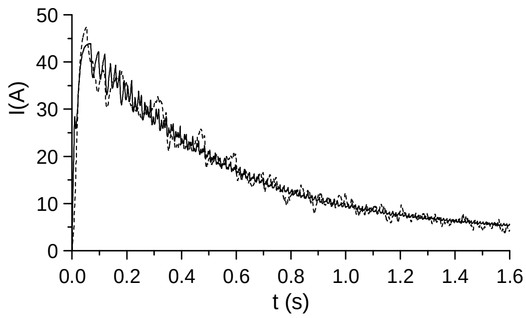

Fig. 4. Comparison of time responses of stator current amplitude (solid line) and its mathematical model (dashed line)

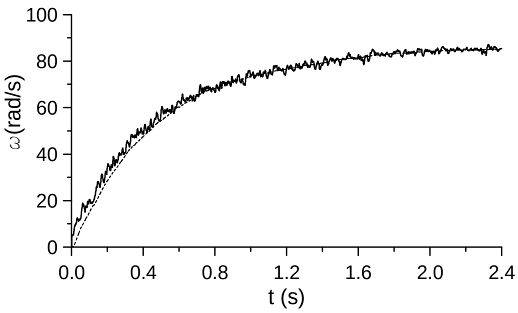

Fig. 5. Comparison of time responses of motor angular velocity (solid line) and its mathematical model (dashed line)

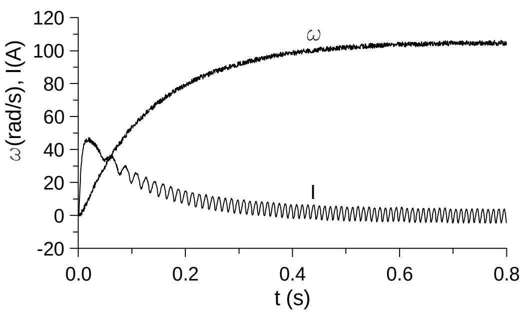

Fig. 6. Step responses of angular velocity and torque

In further discussion, the analysis of angular velocity control dynamics will be made in regulation system with PI type regulator, which block diagram is shown in fig. 7. The algorithm of inverter control was implemented on ADMC 401 microprocessor system (*Analog Devices*). Modulation frequency of pulse widths was assumed as 5 kHz.

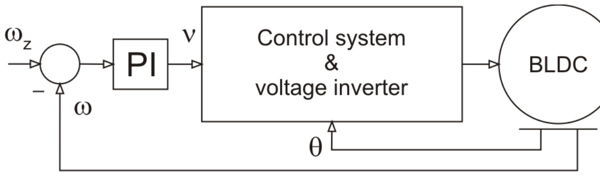

Fig. 7. Block diagram of angular velocity regulation system

In case of drive system with stiff load mass connection in principle, there is no problem to provide the aperiodic damping of angular velocity ω transient process. PI type regulator settings can be determined according to iterative minimization of control performance index [6]

$$Q = \frac{1}{N}\sum_{i=1}^{N}\left(\omega_z(i) - \omega(i)\right)^2 \tag{9}$$

where ω_z – set angular velocity.

Minimization of function (9) leads to the minimization of regulation error and is characterized by slight over-regulation of angular velocity. Rapid achievement of steady state causes high motor current values (fig. 8), and because of high values of regulator gain coefficient – also increased oscillations of current and torque. For the analyzed motor, as a result of minimization of index (9) with Box's numerical optimization method, the following values of regulator parameters were obtained: k_p=218,4 and T_i=0,00077 for $\omega_z = 100$ rad/s.

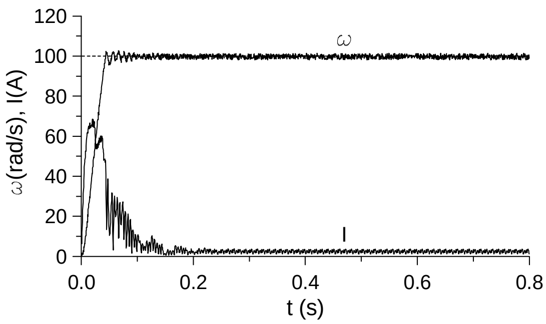

Fig. 8. Time responses of drive system with PI type regulator

For the purpose of forming a time response of angular velocity (a regulation time), the following function can be minimized

$$Q = \left(Q_z - \frac{1}{N} \sum_{i=1}^{N} (\omega_z(i) - \omega(i))^2 \right)^2 \tag{10}$$

where Q_z is the set value, being a mean-square error of motor speed. High value of Q_z provides aperiodic character of transient state with high regulation time (high damping). However, there is often an over-regulation connected with high value of integration constant T_i .

Considerably better result will be obtained by the modification of function (10) by adding a component considering motor current amplitude I

$$Q = \frac{1}{N} \sum_{i=1}^{N} (\omega_z(i) - \omega(i))^2 + \frac{w}{N} \sum_{i=1}^{N} I^2 \tag{11}$$

where the character of transient state is formed by adding an adequate value of weight coefficient w. Step response of motor angular velocity ω and current amplitude I for set angular velocity of $\omega_z = 100$ rad/s is shown in fig. 9, and in this example, the following results were obtained: k_p=1,023 and T_i=6,23.

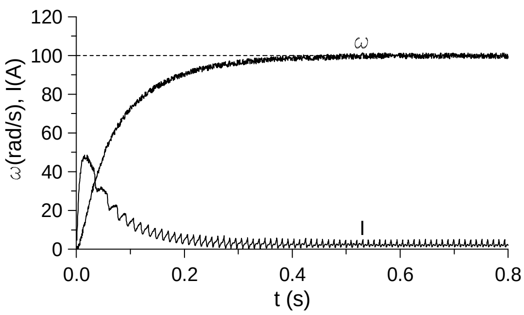

Fig 9. Step responses of angular velocity regulation system

Using the reference model is an effective method of parameter selection. Determination of regulator parameters is made according to a minimization of mean-square error between given time response ω_m (reference model) and time response of regulation system ω

$$Q = \frac{1}{N} \sum_{i=1}^{N} (\omega_m(i) - \omega(i))^2 \tag{12}$$

Reference model equation was assumed in the form of 2nd grade element with the following parameters: gain coefficient K = 1,0, time constant T = 0,02 and relative damping coefficient ξ = 2,0.

Time response of the system during the excitation by set angular velocity ω_z=100 rad/s is shown in fig. 10, and the reference model time response is marked with a dashed line. With the use of PI regulator (k_p=0,936 and T_i=6,57), almost ideal matching of time response of system angular velocity to a reference model characteristic was obtained.

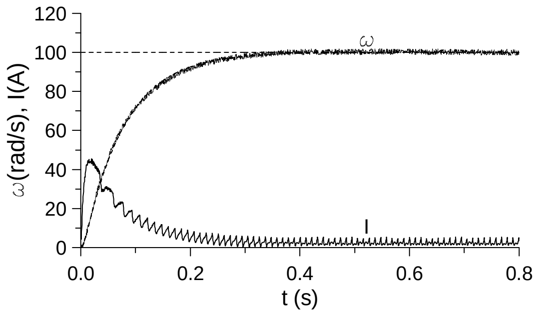

Fig. 10. Time responses of regulation system (solid line) and reference model (dashed line)

Values of motor and load parameters can vary during the operation of power system. Significant feature of every regulation system is its resistance to variations of parameters. Low resistance requires often application of adaptive control. To more precisely determine the influence of variances of particular object parameters on the regulation system time response, sensitivity tests were carried out during transient and steady state. The evaluation of angular velocity regulation system on 20 and 50% value variance of motor parameters was made basing on the following sensitivity index

$$S(i) = \frac{\sum_{i=1}^{N} \omega_p(i) - \sum_{i=1}^{N} \omega(i)}{\sum_{i=1}^{N} \omega_p(i)} \cdot 100\% \qquad (13)$$

where: $\omega_p(i)$ – regulation system response to a set angular velocity value $\omega_z = 100$ rad/s before the change of given parameter, $\omega(i)$ – system response after the change of object parameter. Index calculation was made until the system response reached a steady state with 5% error. System sensitivity was analyzed or the following nominal motor parameters: $L_s = 0{,}0061$ H, $R_s = 1{,}09\,\Omega$, $K_m = 0{,}478$ V/rad, $J = 0.064\,\mathrm{Ns^2/m}$.

Influence of value change of motor parameters to the value of regulation system sensitivity coefficient S in transient and steady state is shown in table 2. According to presented values of sensitivity indexes, practically all analyzed motor parameters influence the transient state, and the induction coefficient K_m and stator resistance R_s have the most considerable influence. Whereas, in case of steady state, only changes of K_m coefficient values have a considerable influence.

Tab. 2. Values of sensitivity coefficients S

Parameter change %	**Transient state**				**Steady state**			
	L_s	R_s	J	K_m	L_s	R_s	J	K_m
20	0,00	0,07	0,02	1,93	-0,02	-0,04	-0,02	0,12
50	0,02	0,10	-0,01	4,76	-0,07	-0,03	0,00	0,37

4. SUMMARY

The subject of this paper was the problem of a drive system velocity regulation with brushless dc motor. It was indicated that the mathematical model parameters of the motor and regulator can be determined according to the application of Box'es numerical static optimization method.

Determination of regulation parameters from the integral quality index minimization condition, which is the function of regulation error, enables to quickly obtain the steady state and quite efficient suppression of oscillation in transient state, but at the cost of excessive values of current and oscillations of motor torque. To have the possibility of forming the unsteady state of regulation system, the modified quality index was used (with the mean-square value of motor current). There was also a proposition of using the reference model and determination of regulator parameters according to the minimization of mean-square error between set (standard) time response and time response of angular velocity regulation system.

Also the analysis of regulation system sensitivity to changes of values of motor parameters was made. It was indicated that changes of motor parameter values influence the character of transient state, whereas they have only a slight influence on the steady state.

5. REFERENCES

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