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INFLUENCE OF FREEPLAY AND FRICTION IN STEERING SYSTEM ON DOUBLE LANE CHANGE MANOEUVRE

Abstract: Sensitivity and simulation investigations of a car dynamics model with regard to freeplay (backlash in steering gear) and friction (friction with stiction in king-pins) is a subject of the paper. The studies deal with a problem of sensitivity of optimized manoeuvres (here double lane change manoeuvres). A criterion function refers to several evaluations (precision of manoeuvre, calm steering, feeling of comfortable travel). The optimization of input steering signal (steering wheel angle) is done on a reference model with „nominal" freeplay and friction parameters. His input signal is applied to a „real" vehicle having „real" freeplay and friction parameters. The differences of output signals as well as differences between criterion function values are a subject of sensitivity analysis. For modelling and simulation of freeplay / friction effects special piecewise-linear luz(…) and tar(…) projections have been used.
Keywords: vehicle dynamics, steering system, freeplay, friction, sensitivity analysis, optimization of manoeuvre, double lane change manoeuvre.

1. INTRODUCTION

Control systems based on computer and mechatronic devices change a car "philosophy" and technology. A typical handling is replaced by automatic control. Note, the ABS, ASR, ESP which enable automatic stabilization of car movements became standard systems in a modern car. We can notice also a big progress of prototype systems designed for automation of maneuvers (e.g. so called assistance steering systems). Such systems take advantage of reference simulation models of a car dynamics, which are executed by a computer in real time ("on-line" simulation), or are based on ready-to-use steering procedures elaborated with "off-line" simulation but activated automatically after detection and identification of road events.

A sensitivity analysis of vehicle system dynamics is a key for synthesis of control algorithms. An effective modeling requires many simplifying assumptions. Model reductions are demanded because of practical and theoretical limitations of identification and simulation procedures especially when they are provided for on-line calculations. This situation is typical also for reference models used in driver assistance systems aiding a correct execution of vehicle manoeuvres. For simplification of calculations of optimized steering input signals (e.g. steering wheel angle) such models are built neglecting many dynamical factors, particularly neglecting a freeplay (clearance, backlash) and a friction (viscous and dry friction with stiction) nonlinearities in steering mechanisms. However, it is well known that both the freeplay and friction are very important attributes of every car steering system, and therefore they are strictly checked when a car passes seasonal examinations (see state of the art paper [1]). So, well done synthesis of control algorithms should be preceded by a formal sensitivity analysis of vehicle dynamic models.

The paper takes up the problems of sensitivity analysis of steering system model applied for synthesis algorithms for a double lane change manoeuvre. Such manoeuvre is typical for avoiding and overtaking. This is continuation of authors' studies presented in [4], [5].

2. THEORETICAL BACKGROUND OF MODELLING AND SENSITIVITY ANALYSIS OF SYSTEMS WITH FREEPLY AND FRICTION

Modelling of multi-body systems (MBS) with freeplay (backlash, clearance) and friction (kinetic and static) provide strong non-linear variable-structure differential equations with algebraic constrains. Such models are difficult for analysis and simulation. A quest of more "friendly" methods of modelling has ever been an attractive challenge.

In cases of steering mechanisms, the freeplay and friction actions can be expressed by piecewise linear models based on piecewise linear characteristics - see fig.1.

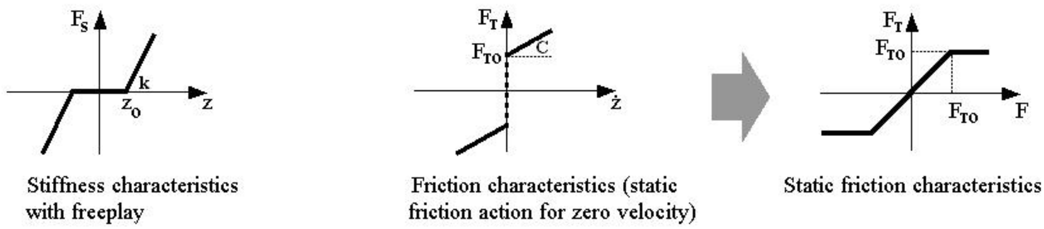


Fig. 1. Typical characteristics for freeplay and friction descriptions. Here is a version for kinetic and static friction force parameter $F_{TK0} = F_{TS0} = F_{T0}$. Notation: k – stiffness coefficient, z_0 – freeplay parameter, C – viscous friction coefficient , F_{T0} – dry friction parameter, F_S – stiffness force, F_T – Friction force, z – relative displacement, $\dot{z}$ –relative velocity, F – acting force.

Such characteristics as well as piecewise linear models (e.g. stick-slip models) can be described analytically by luz(…) and tar(…) projections (1), (2), (fig.2.).

$$\mathrm{luz}(x,a) = x + \frac{|x-a| - |x+a|}{2}, \quad a \geq 0 \tag{1}$$

$$\mathrm{tar}(x,a) = \mathrm{luz}^{-1}(x,a) \qquad \text{(inversion)} \tag{2}$$

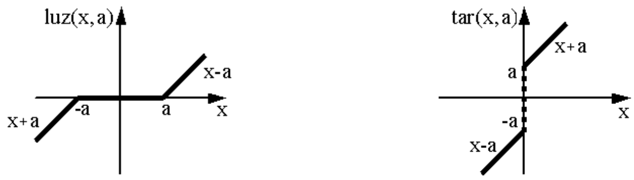


Fig. 2. Topology of luz(…) and tar(…) projections

The tar(...) projection has an indeterminate area for x = 0. This means that models with tar(...) should be treated as inclusion-form models. By additional dependencies, an inclusion description of such model is replaced by variable-structure equations.

Analytical description of freeplay / friction characteristics (from fig.1) are following:

$$F_s = k\,\mathrm{luz}(z, z_0) \tag{3}$$

$$F_T = \begin{cases} C\,\mathrm{tar}\left(\dot z, \dfrac{F_{T0}}{C}\right) & \text{if} \quad \dot z \neq 0 \\ F - \mathrm{luz}(F, F_{T0}) & \text{if} \quad \dot z = 0 \end{cases} \tag{4}$$

The forms with luz(…) and tar(…) projections occur in models of systems with freeplay /friction action, and with stick-slip effects. An original mathematical apparatus (details in Żardecki's monograph [8] and articles, e.g. [7]) is especially useful for simplification and parametricaly-made reductions of such non-linear models.

The luz(…) and tar(…) projections have been used for description of freeplay (gear backlash) and friction actions and stick-slip effects in steering system mechanisms.

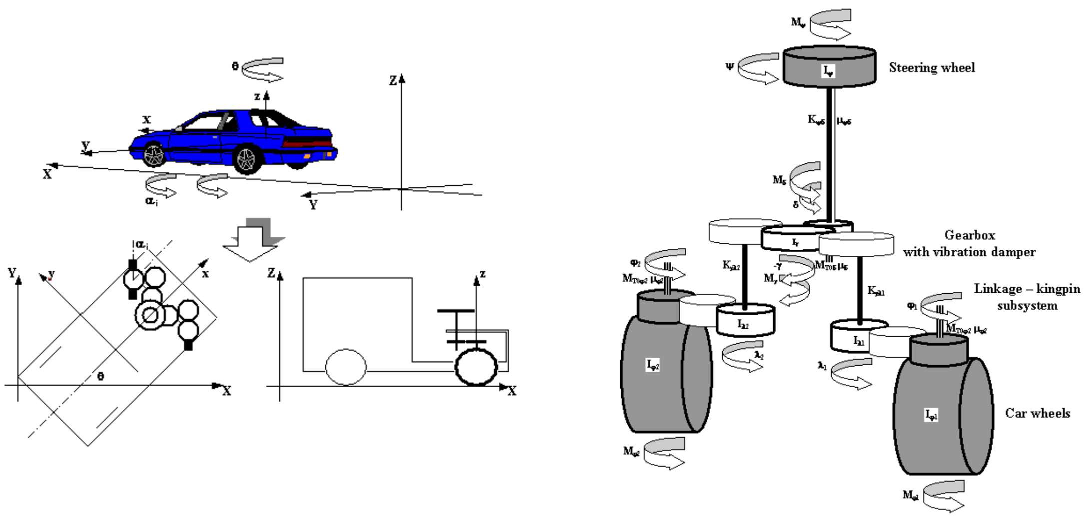


Fig. 3. Conception of MBS-type physical model of steering system mechanizm

Żardecki's monograph [8] contains different models – the most complex MBS primary model (see fig.3.) and several simplified models by parametrically-made reductions, which are suitable for additional assumptions. More complicated models have been used in simulations of different open car tests (ISO and ECE), for different vehicle configuration (2WS and 4WS), different steering system structure (classic or with power assistance), different forms of kinetic friction (Coulomb- and Stribeck-type characteristics) and different parameters (e.g. variations of freeplay and dry friction parameters). The simplest model (5) (single-mass "bicycle" model basing on substitute parameters) is provided for symmetrical construction and excitations. This model is applied in a synthesis of steering signals for driver assistance systems.

$$I_\varphi \cdot \ddot{\varphi}(t) = \begin{cases} -\mu_\varphi \cdot \mathrm{tar}\left(\dot{\varphi}(t), \dfrac{M_{T0K\varphi}}{\mu_\varphi}\right) + M(t) & \text{if} \quad \dot{\varphi}(t) \neq 0 \\ \mathrm{luz}\left(M(t), M_{T0S\varphi}\right) & \text{if} \quad \dot{\varphi}(t) = 0 \end{cases} \tag{5}$$

where $M(t) = p \cdot M_\psi(t) + M_\varphi(t)$, $M_\psi(t) = K_{\psi\varphi} \cdot \mathrm{luz}\left(\left(\psi(t) - p \cdot \varphi(t)\right), (\delta - p\gamma)_0\right)$.

Notation: ψ, φ- angles of steering wheel and steered wheel, I_φ - moment of inertia, μ_φ - viscous friction coefficient, $M_{T0K\varphi}$, $M_{T0S\varphi}$- kinetic and static dry friction parameters, $K_{\psi\varphi}$ - stiffness coefficient, $(\delta - p\gamma)_0$ - freeplay parameter (one half of "dead zone"), p – gear ratio.

Analysis of freeplay / friction influence on a car lateral dynamics can be realized on the model composed with the model of steering system and the model of vehicle motion (fig.4.).

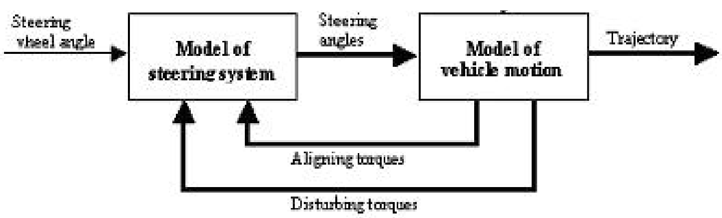


Fig. 4. Conception of decomposition of a car dynamics model into two partial sub-models

From mathematical point of view such analysis can be treated as a parametrically-made sensitivity analysis of the car dynamics model (fig.5.).

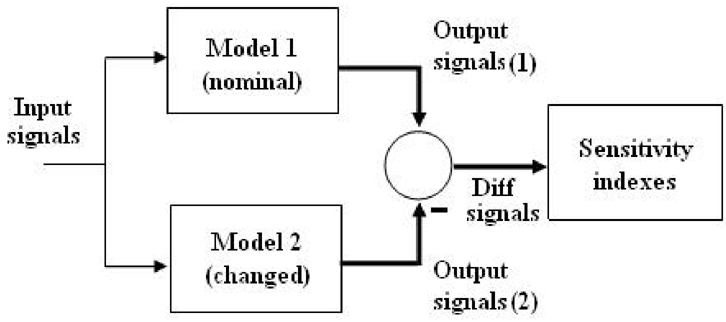


Fig. 5. General schematic diagram of sensitivity analysis

When model equations have regular forms and their changes result from parametrically-made perturbations, sensitivity indexes are continuous functions of these parameters. Classic parametric methods take advantage of a variation analysis, so the continuity and differentiation of model equations is demanded. In our case conditions for differentiation of model equations are not fulfilled (non-smooth model). But the postulate of regularity of the model equations is fulfilled and secured by the luz(…) and tar(…) mathematical apparatus (more details in Żardecki's papers [6], [9]). So difference signals are continue in relation to the freeplay or friction parameters and for a detail analysis the simulation methods can be applied. According to this concept, the sensitivity analysis is provided on the base of many comparative simulations.

In this paper the sensitivity / simulation analysis concerns optimized road manoeuvres based on the simplified reference models.

3. OPTIMIZATION OF DOUBLE LANE CHANGE MANOEUVRE

Optimization of road manoeuvres (for constant vehicle velocity by optimization of steering wheel angle) is a first steep for synthesis a driver assistance system. An idea, a method and results of optimization of steering system input signals for typical road manoeuvres have been presented with details in Wieckowski's dissertation [3] and several papers (e.g. [2]). An outline of this conception is shown below.

The criterion function (6) of such optimisation includes the following evaluations:
- precision of movement track performance – adherence to the defined lane,
- calm steering – avoiding violent, frequent movement of the steering wheel,
- feeling of comfortable travel for passengers.

$$J_w = w_1 \cdot \frac{1}{T}\int_0^T \dot{\psi}(t)^2 \, dt + w_2 \cdot \kappa_{max}^2 + w_3 \cdot a_{y\,max}^2 \qquad (6)$$

where:

κ - inverse of distance between a car and an edge of traffic lane (measure of precision),

$\dot{\psi}$ - steering wheel angular velocity (measure of calm steering),

a_{ymax} - maximal lateral acceleration value (measure of feeling of passenger comfort).

w_1, w_2, w_3 - "weight" coefficients (they are speed-dependent factors).

The task of optimization of manoeuvre is resolved in two stages.

1) Synthesis of a piecewise-sinusoidal / constant form of input signal with unknown values of amplitudes, frequencies and switch times. This is made by a detail analysis of signals registered during a real manoeuvre of the car and supported by simulation studies.
2) Optimization of unknown signal parameters. This includes: finding $\min(J_w)$, and fulfilling limiting conditions – vehicle adherence to a given movement track, and lateral acceleration values to the permissible extent ($a_{ymax} \le 4$ m/s^2). If the limitations are not fulfilled $\min(J_w) = \infty$.

The optimisation task is made by series of simulations based on the reference model.

Exemplification of the method for double lane change manoeuvres (avoiding and overtaking) is showed on fig. 6, 7 and 8.

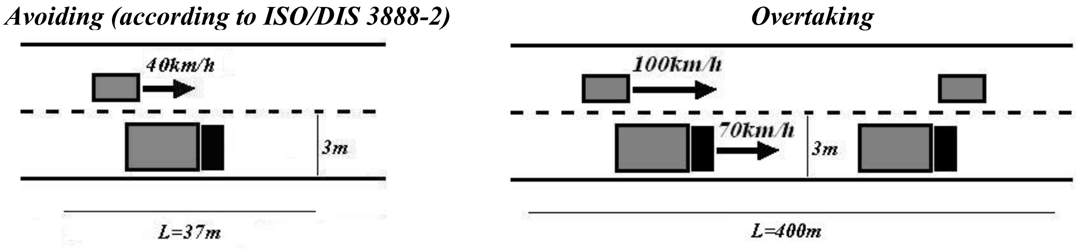


Fig. 6. Idea of double change lane manoeuvre when avoiding or overtaking (L – length of “corridor”)

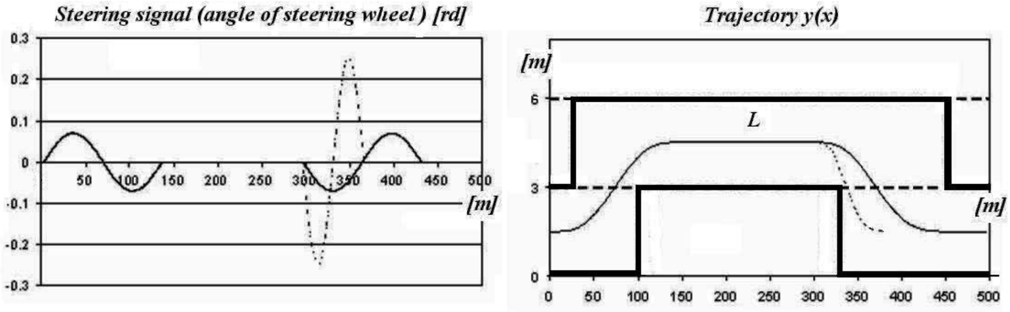


Fig 7. Example results: optimal (continuous line), and another (dash line) – here for overtaking

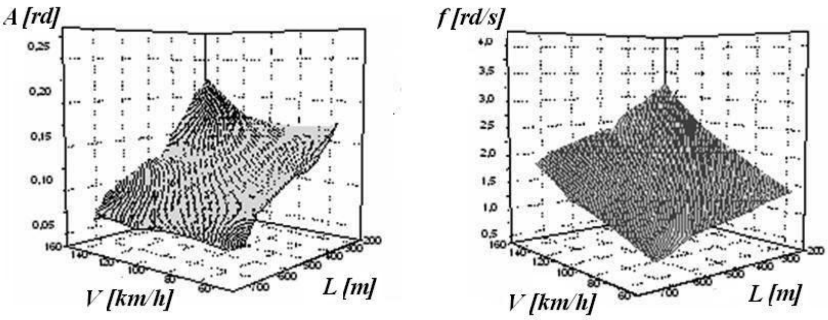


Fig 8. Example dependences of input amplitude A and frequency f upon length of “corridor” L and speed V

In these optimization studies the steering system model has been assumed as a simple one-mass linear system (no freeplay and dry friction effects). The vehicle motion has been described typically for constant speed manoeuvres (lateral and roll dynamics). The full model has had 12 degrees of freedom [3].

The results of optimization give us an “optimal” steering signal $\hat{\psi}(t)$ which is then applied to a real vehicle having a steering system with non-linearities (freeplay in steering gear, and dry fiction with stiction in king-pins). It is very interesting and important to analyse the sensitivity of the results on simplifications of the reference model, especially checking of limiting conditions fulfilled. After all, an adherence to a given movement track is absolutely inadmissible! This can be tested by simulations of real manoeuvres with a non-linear steering system model.

4. SENSITIVITY ANALYZIS OF REFERENCE MODEL OF DOUBLE LANE CHANGE MANOEUVRE

The studies have been organized as follow: Nominal and varied values of parameters $(\delta - p\gamma)_0$ and $M_{T0\varphi}$ ($M_{T0\varphi} = M_{T0\varphi} = M_{T0\varphi}$) are assumed in two sets: $\{(\delta - p\gamma)_0\}$ and $\{M_{T0\varphi}\}$. For every one combimation of the parameters an optimal steering signal $\hat{\psi}_{i,j}(t)$ is calculated. So, the list of variants includes (i_{max} x j_{max}) functions having different amplitudes and frequencies and switching times of sinusoides. For the pair $(\delta - p\gamma)_{0\,i}$ and $M_{T0\varphi\,j}$ we receive the output signals (indexed by i,j) and minimum value $J_{wi,j}$ ($J_{wi,j} = \infty$ when limiting are not fulfilled). Then the signals $\hat{\psi}_{i,j}(t)$ are applied as steering signals to the model with varied parameters (from the sets). The signals and the values of J_w are compared to "optimal" ones. Thank to these sensitivity studies we can find pairs of freeplay / friction parameters $(\delta - p\gamma)_{0\,i}$ and $M_{T0\varphi\,j}$ for which the optimal steering is the most „robust".

Because of editional limitation only short sets have used here: $(\delta - p\gamma)_0 \subset \{0,\ 0.05,\ 0.10\}$rd, $M_{T0\varphi} \subset \{0,\ 4.05,\ 8.10\}$Nm. The zero values should be treated as the „nominal" parameters.

The studies have concerned the avoiding as well as overtaking problems. The example results are showed on fig.9 and 10.

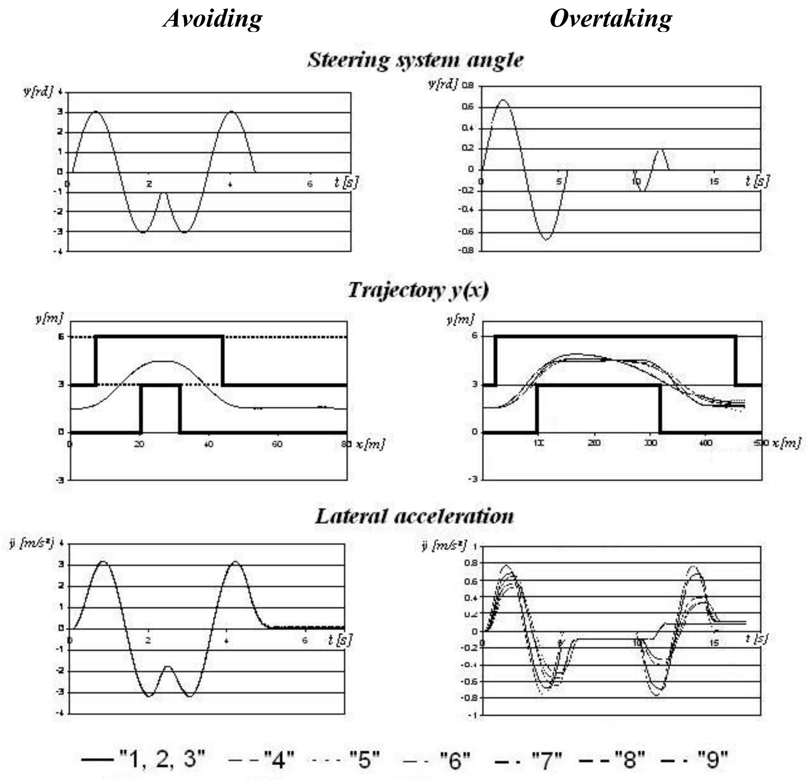
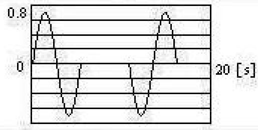
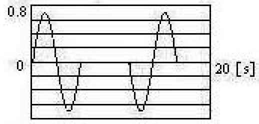
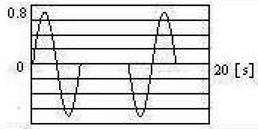
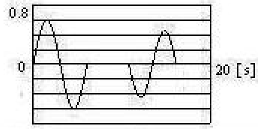
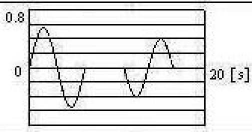
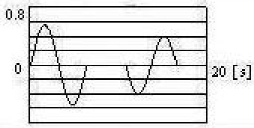
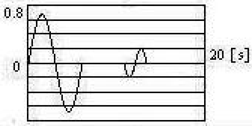
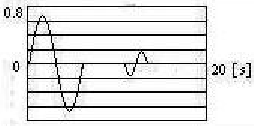
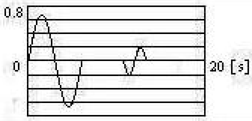


Fig.9. Example simulation / sensitivity results for overtaking. Numbering according to fig.10

Overtaking

No	Freeplay [rd]	Friction [Nm]	Optimal steering signal [rd]	Value of criterion function Jw: Frict \ Free	0.00	0.05	0.10
1	0.00	0.00		0.00	4.659	6.569	32.148
				4.05	4.659	6.017	35.671
				8.10	4.562	6.126	36.454
2	0.00	4.05		0.00	4.659	6.569	32.148
				4.05	4.659	6.017	35.671
				8.10	4.5612	6.126	36.454
3	0.00	8.10		0.00	4.6585	6.569	32.148
				4.05	4.6585	6.017	35.671
				8.10	4.5615	6.126	36.454
4	0.05	0.00		0.00	∞	6.569	∞
				4.05	∞	5.861	∞
				8.10	∞	5.856	∞
5	0.05	4.05		0.00	∞	∞	32.110
				4.05	∞	6.017	∞
				8.10	∞	6.013	∞
6	0.05	8.10		0.00	∞	6.688	∞
				4.05	∞	6.129	∞
				8.10	∞	6.126	∞
7	0.10	0.00		0.00	∞	∞	32.110
				4.05	∞	∞	32.105
				8.10	∞	∞	32.112
8	0.10	4.05		0.00	∞	∞	35.676
				4.05	∞	∞	35.671
				8.10	35.678	∞	32.112
9	0.10	8.10		0.00	∞	∞	36.453
				4.05	4.682	∞	36.448
				8.10	4.562	∞	36.442

Avoiding

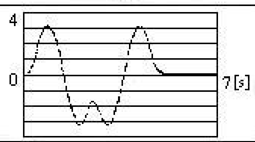
No	Freeplay [rd]	Friction [Nm]	Optimal steering signal [rd]	Value of criterion function Jw: Frict \ Free	0.00	0.05	0.10
1 - 9	0.00	0.00		0.00	32.466	32.506	32.656
				4.05	32.466	32.513	32.668
				8.10	32.469	32.501	32.696

Fig.10. Influence of freeplay / friction parameters on the J_w for different variants of optimized input signals

5. CONCLUSION

The results of studies are summed as follow:

- The optimal steering signal depends upon freeplay / friction parameters especially for longtime manoeuvres (overtaking).
- An influence of the feeplay parametr is evident (for example, when the freeplay increases from 0° to 10°, J_w grows up from 4.859 to 32.148). An influence of the dry friction parameter seems to be passing over. This is rather clear because in these studies the input signal (steering wheel angle) has been a kinematic excitation. In case of dynamic excitation (steering torque) a situation seems to will be different.
- Because of robustness it is recommended to made optimization on the model having zero value freeplay parametr.
- The studies should be continued (more extended excitation, different manoeuvres, different structures of the steering system and the car).

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WPLYW LUZU I TARCIA W UKŁADZIE KIEROWNICZYM NA MANEWR PODWÓJNEJ ZMIANY PASA RUCHU

Streszczenie.: Przedmiotem pracy są badania symulacyjne i analiza wrażliwości modelu dynamiki samochodu z uwagi na luz (w przekładni) i tarcie (tarcie kinetyczne i statyczne w

zwrotnicach). Badania ukierunkowane są na problematykę wrażliwości i odporności dotyczącą optymalizowanego manewru (tu podwójne zmiany pasa ruchu). Funkcja kryterialna uwzględnia kilka czynników (precyzję manewru, spokojne sterowanie, poczucie komfortu podróży). Optymalizacja wejściowego sygnału sterującego (kąt obrotu kierownicy) jest wykonana na podstawie modelu referencyjnego z „nominalnymi" parametrami luzu i tarcia. Obliczony sygnał wejściowy jest zastosowany do „rzeczywistego" pojazdu mającego „rzeczywiste" parametry luzu i tarcia. Różnice sygnałów wyjściowych jak również różnice pomiędzy wartościami funkcji kryterialnej są przedmiotem analizy wrażliwości. W modelowaniu i symulacji efektów luzu i tarcia wykorzystano specjalne, przedziałami liniowe odwzorowania luz(…) i tar(…).
Słowa kluczowe: dynamika pojazdu, układ kierowniczy, luz, tarcie, analiza wrażliwości, optymalizacja manewrów drogowych, manewry podwójnej zmiany pasa ruchu